Robust Subspace Estimation Using Low-Rank Optimization

SHORT, PARTIAL BOOK SUMMARY

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Summary

- Use Alternating Directions Method of Multipliers (ADMM) Method Augmented Lagrange Multiplier (ALM) to solve thee problems:
  - Underwater Scene Reconstruction
    - Perform robust registration on frames
    - Decompose into low rank and sparse components $\arg\min_{A,E} rank(A)$ s.t. $F = A + E$ and $||E||_0 \leq \beta$
  - Simultaneous Turbulence and Motion
    - Treat the turbulence as Gaussian white noise and solve $\arg\min_{A,O,E} rank(A)$ s.t. $F = A + O + E$ and $||O||_0 \leq s,\ ||E||_F \leq \sigma$
    - With moving object tracking to help improve discrimination of moving objects and turbulence
  - Action and Event Recognition (not covered)

Methodology

- Paper References

  Z. Lin, M. Chen, L. Wu, and Y. Ma. The augmented lagrange multiplier method for exact recovery of corrupted low-rank completion. In UIUC Technical Report, 2009.
  Code: http://perception.csl.illinois.edu/matrix-rank/home.html

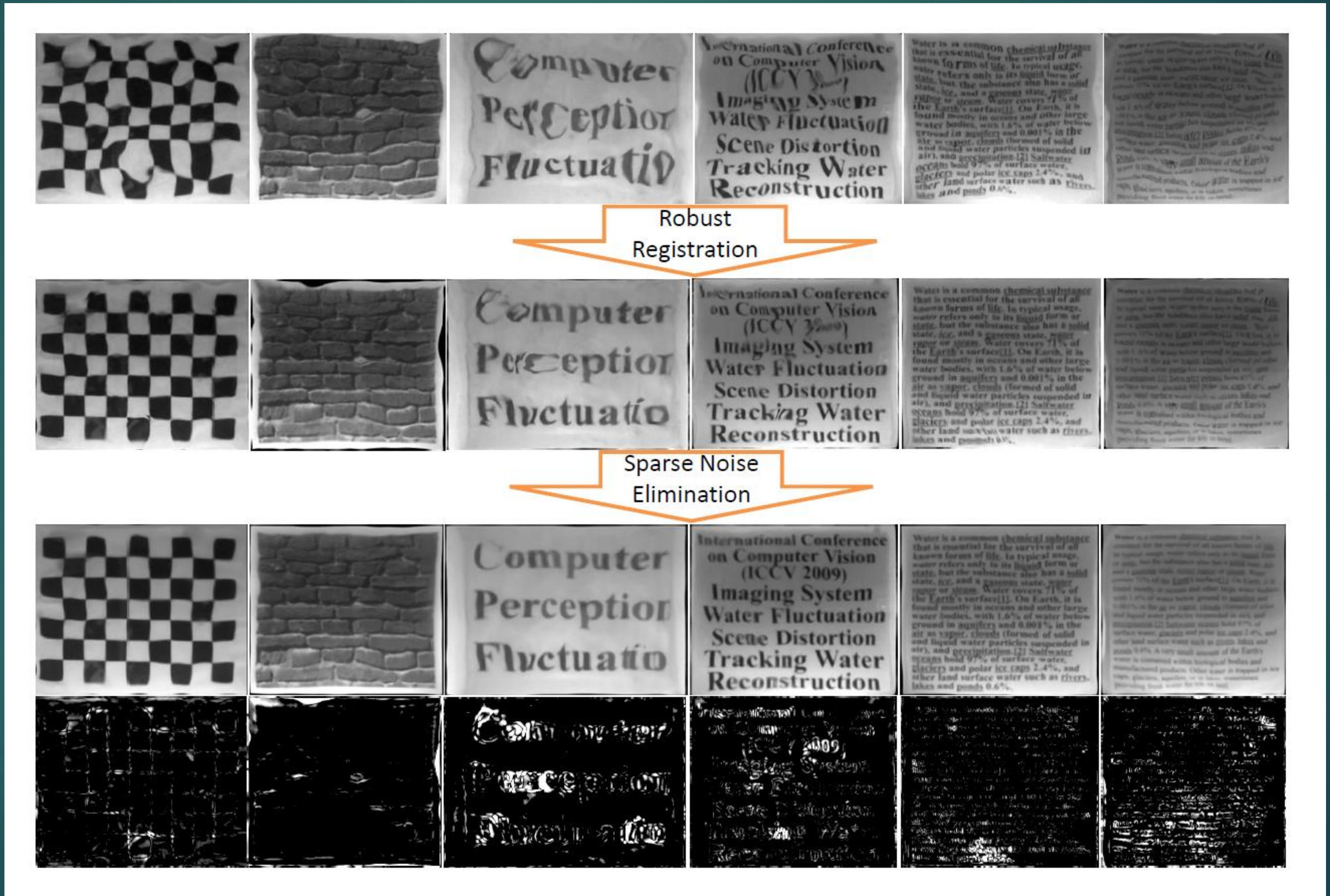
  That claims to be *five times* faster than Accelerated Proximal Gradient (APG) method. Same method used in Candes PCA?

  Candès, E. J., Li, X., Ma, Y., & Wright, J. (2011). Robust principal component analysis?. *Journal of the ACM (JACM)*, *58*(3), 11.

Underwater Scene Reconstruction

- Robust Registration
  - Create reference image from mean
  - Blur Frames similar to reference and use Multiscale B-Spline to register all frames to reference
  - Use mean of registered image as new reference
  - Repeat Blur Frames similar to new reference and multiscale B-Spline registration to new reference. Repeat further as needed.
- Low Rank Recovery
  - Decompose vectorized frames F into Low Rank and Sparse Error
    $\arg\min_{A,E} rank(A)$ s.t. $F = A + E$ and $\|E\|_0 \leq \beta$
  - With convex relaxation
    $\arg\min_{A,E} \|A\|_* + \lambda\|E\|_1$ s.t. $F = A + E$

Example Results



Simultaneous Turbulence and Motion

- Treat the per pixel turbulence fluctuations E as white Gaussian noise, the moving objects O as sparse, and the background A as low rank and solve

  $$\arg\min_{A,O,E} rank(A) \text{ s.t. } F = A + O + E \text{ and } \|O\|_0 \leq s, \|E\|_F \leq \sigma$$

  through the convex relaxation

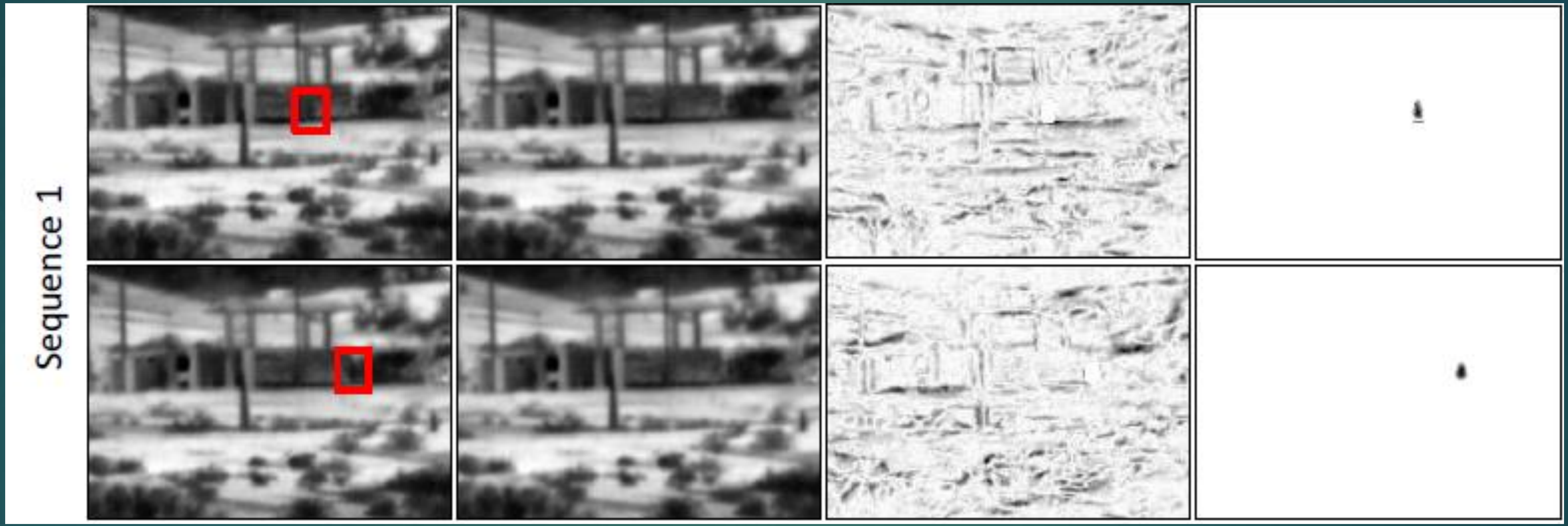
  $$\arg\min_{A,O,E} rank(A) \text{ s.t. } F = A + O + E \text{ and } \|\Pi(O)\|_0 \leq s, \|E\|_F \leq \sigma$$

- Because the turbulence motion tends to appear in the moving object motion, Π is introduced as a certainty weighting to threshold out uncertain objects. A fair amount of work is done to try and create the weighting by tracking the particle displacement of objects and describing them as consistent with the turbulence model or not

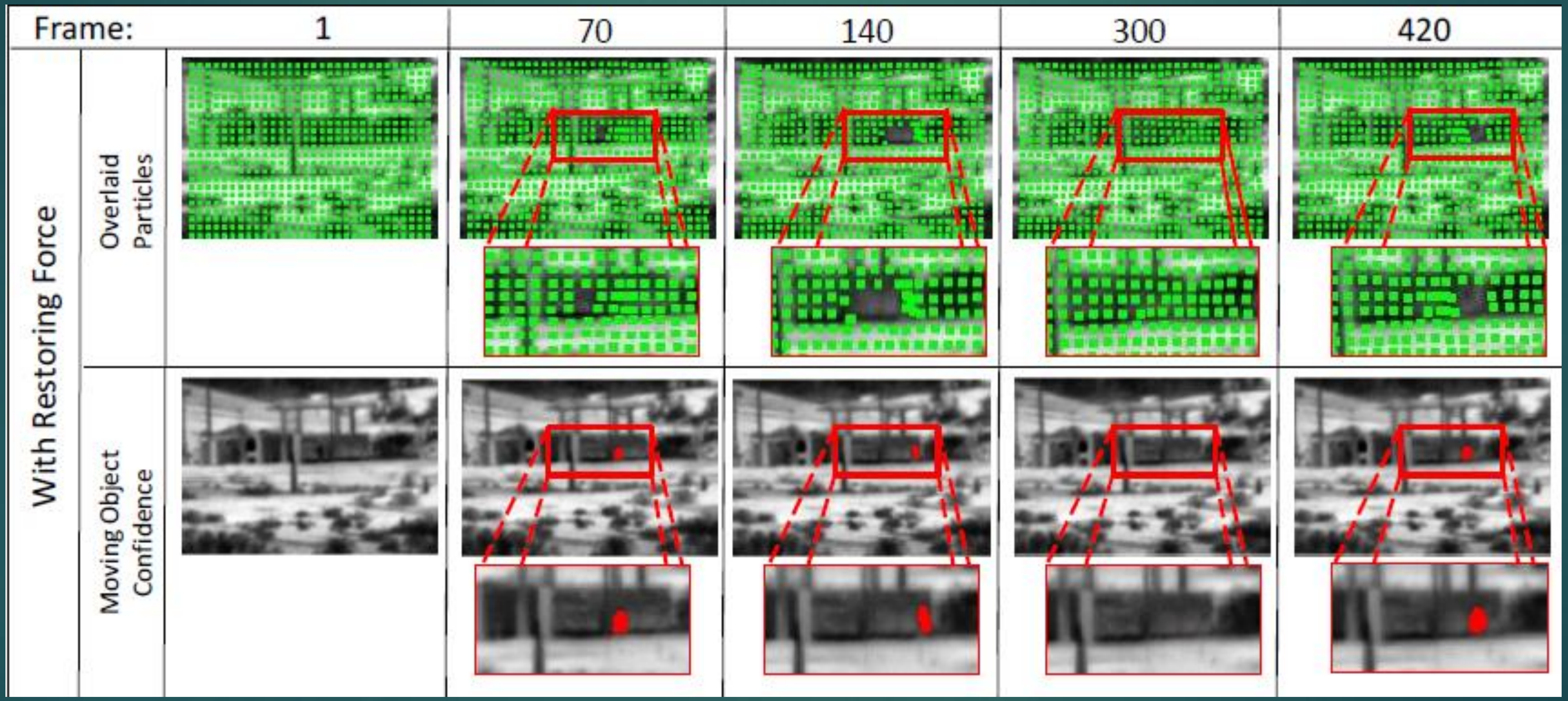
Sketch of Formulation

- Try convex relaxation $\arg\min_{A,O,E} \|A\|_* + \tau\|\Pi(O)\|_1 + \lambda\|E\|^2{}_F$ s.t. $F = A + O + E$
- With Augmented Lagrange Function $L(A, O, E, Y) = \|A\|_* + \tau\|\Pi(O)\|_1 + \lambda\|E\|_F^2 + < Y, F - A - O - E > + \frac{\beta}{2}\|F - A - O - E\|_F^2$ Where Y is a Lagrange multiplier matrix, β is a positive scalar, and <,> denotes matrix inner product $trace(A^T B)$
- Use ALM to minimize the AL function
  - $(A_{k+1}, O_{k+1}, E_{k+1}) = argmin_{A,O,E} L(A, O, E, Y_k)$
  - $Y_{k+1} = Y_k + \beta_k(F_{k+1} - A_{k+1} - O_{k+1} - E_{k+1})$
  - But this is hard so minimize ALF wrt each component separately (p41/42)
    - $A_{k+1} = \arg\min_{A,O,E} L(A_k, O_k, E_k, Y_k)$
    - $O_{k+1} = \arg\min_{A,O,E} L(A_{k+1}, O_k, E_k, Y_k)$
    - $E_{k+1} = \arg\min_{A,O,E} L(A_{k+1}, O_{k+1}, E_k, Y_k)$
    - With closed form solution worked in appendix

Results



Particle Tracking



References

- 1. O. Oreifej and M. Shah, Robust Subspace Estimation Using Low-Rank Optimization Theory and Applications, Springer, 2014.
- 2. O. Oreifej, "Robust Subspace Estimation Using Low-Rank Optimization. Theory And Applications In Scene Reconstruction, Video Denoising, And Activity Recognition," University of Central Florida, 2013.
- 3. Z. Lin, M. Chen, L. Wu, and Y. Ma. The augmented lagrange multiplier method for exact recovery of corrupted low-rank completion. In UIUC Technical Report, 2009.